

# An Introduction to MIMO OFDM System with BER Analysis

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**Abstract:** OFDM is a frequency-division multiplexing (FDM) system used as a digital multi-carrier modulation method. A large number of closely spaced orthogonal sub-carrier signals are used to carry data on several parallel data current or channels. In this paper, BER performance of MIMO (Multi-Input Multi-Output) OFDM system has been compared for AWGN channel, Rayleigh fading channel and Rician fading channel. Later analyze how with different value of Diversity parameter BER performance of Rayleigh fading channel and Rician fading channel has been effected. This paper has been divided into two parts. In first part basic function of OFDM has been introduced and then in second part BER performance of OFDM has been analyzed.

**Keywords:** IFFT, Cyclic prefix, ISI, AWGN, Rayleigh, Rician, BER, Matlab, MIMO, OFDM.

## I. INTRODUCTION

OFDM is a popular technique for wireless high data rate transmission, because it enables efficient use of the available bandwidth and a simple implementation. It divides the frequency selective fading channel into parallel flat fading sub-channels. OFDM is a broadband transmission technique i.e. having larger bandwidth. Let bandwidth of the signal is 'B'. So, for single carrier, symbol time period (1/B) will be higher than delay spread of channel which results in inter-symbol interference (ISI) [1].

There are various adaptive channel estimation methods have been developed which tries to minimize error [2...8]. To avoid ISI, divide whole bandwidth into smaller bandwidth i.e. called sub-band. Each sub-band has a subcarrier. Let the number of sub-band be 'N'. Since bandwidth of each sub-band is B/N, symbol time period is greater than delay spread of channel, hence no ISI. Such a system with no multiple sub-bands and multiple sub-carriers is termed as Multicarrier modulated (MCM) system. MCM system is the basis for OFDM.

## II. TRANSMISSION IN MCM SYSTEM

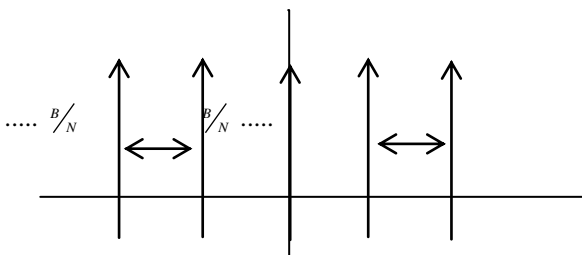


Fig. 1. Bandwidth of MCM system

In MCM system, there are multiple subcarriers with spacing B/N. Take fundamental frequency as  $f_0=B/N$ .  $k_{th}$  subcarrier which is centered at  $kf_0=kB/N$  is given as  $e^{j2\pi kf_0 t}$  with symbol  $X_k$ . Then signal transmitted on  $k_{th}$  subcarrier is:

$$S_k(t) = X_k e^{j2\pi kf_0 t} \tag{1}$$

Net transmitted MCM signal, which is the sum of transmit signals across all 'N' subcarriers  $= \sum_k X_k e^{j2\pi kf_0 t}$ .

Assume that there is no noise component, and then the received signal is  $Y(t) = \sum_k X_k e^{j2\pi kf_0 t}$

Above equation is similar to Fourier series. Now to recover this symbol, let the symbol on  $l_{th}$  subcarrier be  $X_l$  and to extract  $X_l$  using inverse Fourier transform (IFT)

$$IFT = f_0 \int_0^{1/f_0} e^{-j2\pi f_0 t} y(t) dt = \sum_k X_k f_0 \int_0^{1/f_0} e^{j2\pi (k-l) f_0 t} dt$$

$$\text{where } f_0 \int_0^{1/f_0} e^{j2\pi (k-l) f_0 t} dt \begin{cases} = 1, & \text{if } k = l \\ = 0, & \text{if } k \neq l \end{cases}$$

$$IFT = \sum_k X_k \delta(k-l) = X_l \tag{2}$$

So at receiver employ coherent demodulation with  $e^{-j2\pi f_0 t}$  to extract  $X_l$ . By correlating with  $e^{j2\pi f_0 t}$  or using matched filter with  $e^{-j2\pi f_0 (T-t)}$ , one can extract symbol on  $l_{th}$  subcarrier. This is the principle of MCM system.

### A. Single Carrier and Multi-Carrier System

In single carrier system with symbol time of  $1/B$  which is used for single subcarrier, system transmits N symbols in time  $N/B$ . While in MCM system with N subcarriers symbol time is  $N/B$ , so here symbol time increases and transmitting 'N' symbol in parallel across 'N' subcarriers. So each symbol has time  $N/B$ . Since MCM is a parallel transmission system as parallel channel between

transmitter and receiver; each of bandwidth  $\frac{B}{N}$ , each of symbol time  $\frac{N}{B}$  and 'N' such sub-band used with 'N' subcarrier to convey 'N' symbols in parallel between transmitter and receiver.

**B. FFT and Cyclic Prefix for OFDM**

Two key aspects of OFDM are Inverse FFT (IFFT) / FFT processing and cyclic prefix:

• IFFT/FFT processing:

Consider the MCM signal  $x(t) = \sum_k X_k e^{j2\pi k f_0 t}$  generating

this signal is difficult because of large number of sub-carriers. In this system, each subcarrier need an oscillator and these oscillators have to be precisely placed at that subcarrier spacing at  $\pm f_0, \pm 2f_0 \dots$ , because only then orthogonality principle between these subcarrier will be hold. To overcome the problem of large number of subcarrier uses the property of signal of being band-limited. Since the signal is band-limited to  $f_{max}$ , it can be sampled at the Nyquist rate of '2f<sub>max</sub>' which is equal to 'B'.

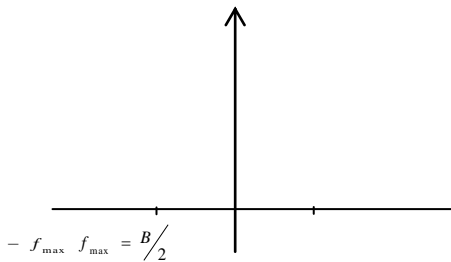


Fig. 2. Bandlimited signal which is sampled at Nyquist rate

As sampling rate =  $\frac{1}{\text{sampling frequency}} = \frac{1}{B} = T$

Since 1<sub>th</sub> sampling interval =  $IT = \frac{l}{B}$

Now,  $x(t) = \sum_k X_k e^{j2\pi k f_0 t}$

1<sub>th</sub> sample  $x(l) = x(IT) = \sum_k X_k e^{j2\pi \frac{k l}{N}}$  (3)

Above equation resembles IDFT or FFT. So, eq (3) is 1<sub>th</sub> point of symbols  $X_0, X_1, X_2, \dots, X_{N-1}$ . So here we don't need to fulfill the requirement of oscillators or precise multi-carrier spacing, if we consider samples of transmit signal. These samples can be generated by the simple IFFT operation. So IFFT at transmitter and FFT at receiver is the key step of OFDM. In MCM system all symbols are generated using modulators and demodulators, while in OFDM these modulators and demodulators are replaced with IFFT and FFT respectively.

• Cyclic Prefix in OFDM

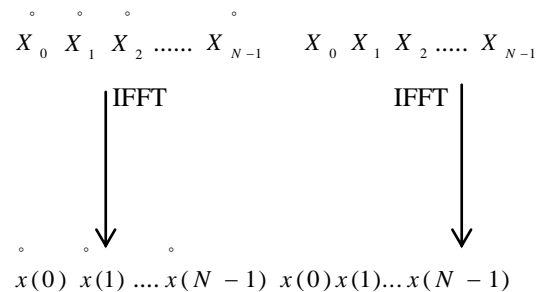
Let's take frequency selective channel, then received signal

$y(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \dots + h(L-1)x(n-L+1)$  (4)

where L is channel taps

From above equation at time instant 'n', y(n) depends on n, n-1,.... so on. So this channel has ISI in time domain and frequency selective fading in frequency domain. Therefore this model represents frequency selective channel.

Considering transmission of two consecutive OFDM symbols:

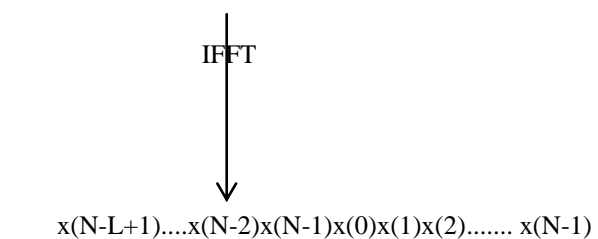


There are two OFDM symbols, output for x(0) of second symbol:

$y(0) = h(0)x(0) + h(1)x(N-1) + h(2)x(N-2) + \dots + h(L-1)x(N-L+1)$  (5)

From output equation, can predict that there are samples from previous block, so there are inter block interference (IBI). To avoid IBI, cyclic prefix will be used i.e.

$X_0, X_1, X_2, \dots, X_{N-1}$



Here last (L-1) samples of transmitted symbols  $x(0)x(1)\dots x(N-1)$  is added as prefix in each OFDM symbol which result in increase of its length to (N+L-1).

Now,  $y(0) = h(0)x(0) + h(1)x(N-1) + h(2)x(N-2) + \dots + h(L-1)x(N-L+1)$

$y(1) = h(0)x(1) + h(1)x(0) + h(2)x(N-1) + \dots + h(L-1)x(N-L+1)$

Similarly get the all other output. Addition of cyclic prefix has resulted in a circular convolution at the output of OFDM system. So the output of OFDM signal is a circular convolution between channel coefficient h(n) and transmitted samples x(n) i.e.

Received OFDM system is

In time domain:  $y(n) = h(n) \otimes x(n)$

In frequency domain:  $Y(k) = H(k) \cdot X(k)$

Using discrete Fourier transform (DFT)/ FFT, get the equation in frequency domain. Here,

(1) Y(k) is k<sub>th</sub> DFT/FFT coefficient of OFDM received signal [y(0)y(1)y(2) ... y(N-1)]

(2) H(k) is k<sub>th</sub> DFT/ FFT coefficient of N-point FFT of [h(0)h(1)h(2)...h(L-1) 000...00], where h(n) are zero padded of length (N-L)

(3)  $X(k)$  is  $k_{th}$  DFT coefficient of  $[x(0)x(1)x(2) \dots x(N-1)]$

Therefore,  $Y(k)$  which is a symbol on  $k_{th}$  subcarrier is multiplication of channel coefficient of  $k_{th}$  subcarrier  $H(k)$  and symbol on  $k_{th}$  subcarrier  $X(k)$ . Since each subcarrier is experiencing 'Flat Fading' and there is no frequency selectivity, which results in no ISI. So with this IFFT/FFT and addition of cyclic prefix method we are able to convert this wireless communication system into frequency flat fading channel across each subcarrier.

### III. BER ANALYSIS OF OFDM SYSTEM

The performance of a communication system can be analyzed in term of Bit Error Rate (BER). BER is a convenient metric which can be employed to characterize the performance of a communication system. Attenuation can cause poor operation in a communicating system because it can effect in a loss of signal power without reducing the power of the noise. This signal loss can be over some or all of the signal bandwidth. Fading can also be a problem as it changes over prison term: communication arrangements are often designed to adapt to such impairments, but the attenuation can change faster than the version can be shuffling.

Here we are summarizing effect of fading in terms of static and dynamic fading [9]. Flat slow fading and flat fast fading are two types of static fading. Frequency selective fast fading and frequency selective slow fading comes under the category of dynamic fading.

One of the simplest forms of fading communication channel model is Additive White Gaussian Noise (AWGN). In this channel, transmitted signal gets attenuated in the path and received signal is the sum of transmitted signal and the noise. Rayleigh fading is a statistical theoretical account for the effect of a multiplication environment on a radio signaling. The model of Rayleigh attenuation channel follows Rayleigh dispersion. It is more effective in the environment when there are so many obstacles in the way of life that is more sprinkling in the way. When there is a dominant allele non-fading signal component present, the small-signal fading envelope is described by a Rician fading. When there is a dominant non fading signal component present, the small signal fading envelope is described by Rayleigh fading. Typically, fading cognitive operation is characterized by a Rayleigh distribution for a non-line-of-sight path and a Rician distribution for a line of sight [10]. A Rician fading channel can be described by two parameters:  $K$  and  $\Omega$  [11].  $K$  is the ratio between the power in the direct path and the power in the other, scattered, paths [12].  $\Omega$  is the total power from both paths, and acts as a scaling factor to the distribution. It is interesting to note that Rayleigh fading channel is a special case of shadowed Rician fading channel. [13]

### IV. RESULTS

The BER performance of OFDM system has been analyzed in different channel i.e. AWGN channel, Rayleigh fading channel and Rician fading channel. Here in our simulation, assuming BPSK modulation technique

and using software Matlab. Fig 3a shows BER performance for different fading channel. From figure it is clear that for BPSK modulation scheme, SNR requirement for AWGN channel is less compare to Rayleigh and Rician fading channel. Here assuming unity diversity order for both fading channel. In figure it can be seen that Rayleigh and Rician fading model are overwriting each other i.e., both showing approximately same characteristics.

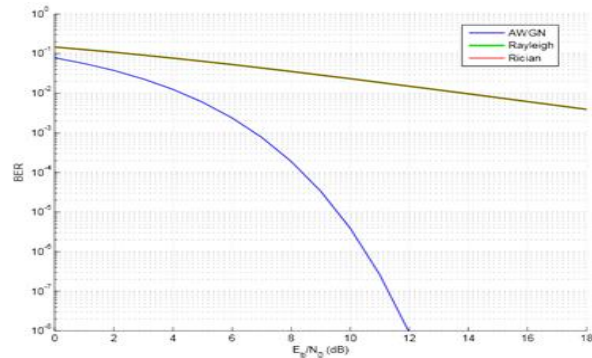


Fig. 3a. BER performance of AWGN, Rayleigh and Rician channel

From fig 3b and 3c both showing same characteristics with unity diversity order and in Rician fading channel assume  $K$ -parameter value as 0 for all reading of fig 3a and 3b. First in fig 3b taking Rician fading channel with perfect synchronization, and analyze BER performance. Next in fig 3c taking Rayleigh fading channel which give the same result as Rician fading channel.

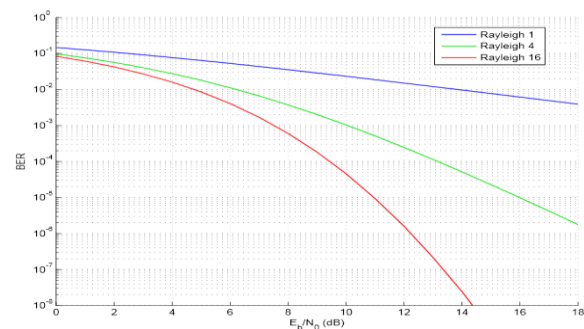


Fig. 3b. BER performance of Rayleigh fading channel for Diversity parameter as 1, 4 and 16 respectively

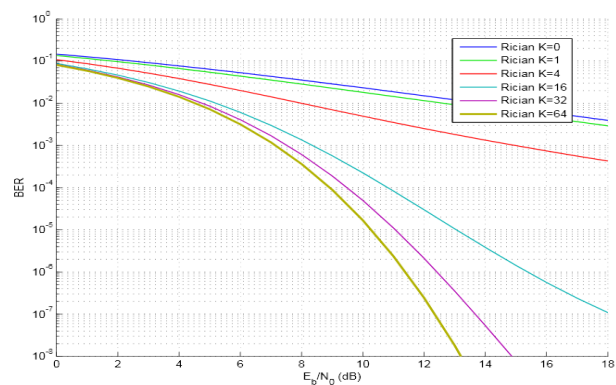


Fig. 3c. BER performance of Rician fading channel for  $K$  value as 0, 1, 4, 16, 32 and 64 respectively

Now analyze the effect of K-parameter on Rician fading channel and keeping the Diversity parameter value fixed to unity. From fig 3c it has been examined that even though by keeping low value of Diversity parameter, with the increment of value of K-parameter requirement of SNR has been decremented and BER performances has been improved.

## V. CONCLUSION

The result analysis shows that initially with default values of Rician and Rayleigh fading channel, AWGN shows better performance comparatively. The results of Rician and Rayleigh fading shows that with increment of Diversity parameter, BER performance has been improved. As Rayleigh fading channel is a special case of shadowed Rician fading channel, with keeping K parameter of Rician fading channel we got same BER performance for both the fading channel. However, there is further possibility of improving BER performance of the Rician fading channel. Even though keeping Diversity parameter at low value, by varying K-parameter can decrease required value of SNR and can have better BER performance.

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